Nucleation of oscillons

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This study concerns the birth of oscillons, the subharmonic localized excitations observed in vibrated sand. To this end, we have constructed a dynamical model based on nearest pattern interaction approximation. The model admits an oscillon solution and features its nucleation process. It is argued here that oscillons are created through nucleation rather than growing from linearly unstable modes. The physical implication of the nucleation in pattern formation is discussed briefly.
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This paper addresses the novel localized subharmonic excitations called ''oscillons,'' discovered recently in vibrated sand [1,2]. To understand the nature of the oscillons, researchers have taken two different approaches. One is the direct molecular-dynamics–type simulations of many interacting particles on a vibrating plate [3]. The other approach has been to model the dynamics of the vibrated granular layer using heuristic or hydrodynamic-type equations [4–7]. Recently, a model discrete in time but continuous in space has been proposed, which is free of physics specific to granular materials [8]. The experimentally observed features of an oscillon are generally reproduced by the model studies [4–6,8]. As it is, it is now more convincing that the experimentally observed oscillons in vibrated sand are not peculiar to the granular materials but rather general in other extended media too. In this regard, we notice that broadly similar structures have been also found in vibrated liquids [9,10].

In this study, we are concerned with the birth of oscillons as it is believed to bear the clues for a variety of global patterns associated with the oscillons. To this end, we need to construct a dynamical model particularly suitable for our purpose. In the absence of any deterministic equations of motion for the granular systems [11], we take the following steps to build our model for oscillons.

Poincaré first recognized the importance of studying the dynamical behavior of mappings, as defined by a difference equation,

$$F:(X_i,\dot{X}_i)\rightarrow(X_{i+1},\dot{X}_{i+1}), \tag{1}$$

where F is a nonlinear transformation [12]. The index i refers to discrete steps in time. Such a mapping is believed to be of general relevance to *differential equations*. For a periodically forced oscillator, the time step corresponds to the period of forcing.

In this study, we extend this methodology to a spatially extended system subject to a periodic forcing [8], and consider the following type of map:

$$F:(h[\vec{r},t],\dot{h}[\vec{r},t])\rightarrow(h[\vec{r},t+T],\dot{h}[\vec{r},t+T]), \tag{2}$$

where the scalar field $h(\vec{r})$ may represent the height of water surface or granular layer at position $\vec{r}$ in a two-dimensional space.

Let us consider in this study only those systems in which the variable $\dot{h}$ is not important in the formation of patterns. For a granular system shaken periodically, for example, there is a periodic collision between sand and container. At the collision stage, information regarding $\dot{h}$ on the velocities of particles would be constantly refreshed. It has been argued that such characteristics may decouple $\dot{h}$ from h in the dynamics of pattern formation in vibrated sand [4,7].

To construct a dynamical model, we take the following two assumptions: (i) A local excitation organizes itself only through interacting with neighboring excitations. (ii) A rise $[h(\vec{r},t)>0]$ or a fall $[h(\vec{r},t)<0]$ from the flat surface $[h(\vec{r},t)=0]$ is equally likely over time.

Nonlinear coupling of the heights between any two locations, say $\vec{r}$ and $\vec{r}\,'$, is believed to be described by a function of the amplitude difference between the two points, say, $\Delta h=h(\vec{r}\,',t)-h(\vec{r},t)$. In this study we assume that $h(\vec{r})$ is coupled with those only in some specific neighboring region determined by the size of the characteristic local excitations of the system. To define the neighboring region, let us assume that the domain size of an excitation is represented by a circle with radius R. The specific neighboring region of the position $\vec{r}$ is then chosen as the region concentric between the circles of radius R and $3R$ centered at $\vec{r}$ as illustrated in Fig. 1(a), where the circles A and A' are inserted to indicate the domain size of the excitations.

To define the term ''nearest pattern interaction approximation'' used in this study, we consider the average fluctuation difference between $h(\vec{r},t)$ and $h(\vec{r}\,',t)$ at the points within the shaded region, defined as

$$\overline{\Delta h(\vec{r},t)}\equiv\frac{1}{\Delta S}\int W_{\vec{r}\,'\vec{r}}[h(\vec{r}\,',t)-h(\vec{r},t)]d^2\vec{r}\,', \tag{3}$$

where $\vec{r}\,'$ now runs over entire space but $W_{\vec{r}\,'\vec{r}}$ is a weighting function being 1 if $R\leqslant|\vec{r}\,'-\vec{r}|\leqslant 3R$ but 0 otherwise, and ΔS denotes the area of the concentric region where $W=1$. We

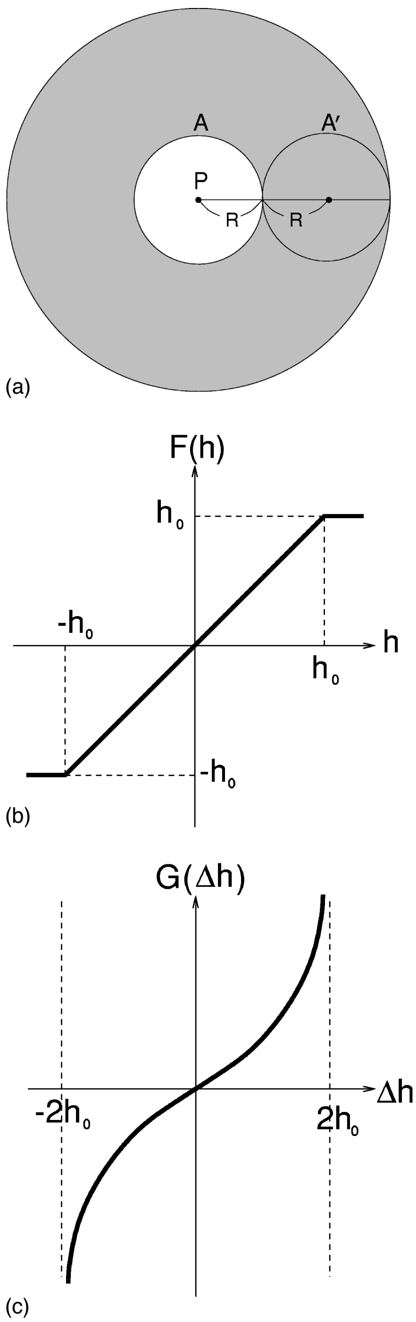

FIG. 1. (a) The field at P is coupled with the fields at other locations only within the shaded region; (b) the function F; (c) the function G.

assume that $h(\vec{r},t)$ is coupled with its neighbors in terms of this averaged fluctuation $\overline{\Delta h(\vec{r},t)}$, and this is our definition of the term ''nearest pattern interaction approximation'' used in this study.

With the definition above, and denoting nT by n, we propose the following dynamical model for oscillons:

$$h_{n+1}(\vec{r})=F[h_n(\vec{r})+\alpha G(\overline{\Delta h_n(\vec{r})})], \quad n=1,2,\ldots, \tag{4}$$

where the function F is a discrete map in time and G is meant to incorporate the interaction between patterns by being a nonlinear function of the averaged fluctuation difference $\overline{\Delta h_n(\vec{r})}$. We apply the one-dimensional map F to the current field $h_n(\vec{r})$ at each location in space modified by $\alpha G(\overline{\Delta h_n(\vec{r})})$ to determine the field $h_{n+1}(\vec{r})$ over the entire space at next time step.

The assumptions (i) and (ii) lead us to choose the specific functional forms of F and G as shown in Fig. 1(b,c). They both are odd functions owing to assumption (ii).

First, F is set as follows with subsequent explanations:

$$F(h)=\begin{cases} h & \text{for } |h|\leqslant h_0, \\ h_0 & \text{for } h>h_0, \\ -h_0 & \text{for } h<-h_0. \end{cases} \tag{5}$$

Assumption (i) has led us to consider a linear map $F(h)$ with a slope of 1 with respect to h in its argument. The slope of F being 1, the field $h(\vec{r},n)$ cannot grow unless it is coupled with ''neighbors.'' The coupling is effected in this study by the function G through the coupling constant α. It is the characteristic of this model that only nonlinear interaction with ''neighbors'' takes part in the process of self-organization of any excitation to arise. Also, since the amplitude of an excitation cannot grow indefinitely, F needs to be bounded. We bound it by some value h_0, which may represent a saturated maximum amplitude of the excitation to arise, if any. In reality, such saturation may be related to mass conservation and/or coarse graining effect. A detailed local structure at the peak of an oscillon is not our interest in this study and thus we simply set the amplitudes higher than h_0 to be just h_0.

For G, we choose a tangent function such as

$$G(\Delta h)=\tan\frac{\pi}{2}\Delta h/(2h_0), \tag{6}$$

where the fluctuation difference Δh is scaled by $2h_0$. Notice that the functional form of F implies that the maximum variation of Δh is $2h_0$, which corresponds to the amplitude difference between the peak and the crater of an excitation.

We choose the above specific forms for F and G because they are the simplest, meeting the basic assumptions for a model. Other nonlinear functions which meet the assumptions will not affect the results while some of the transition points may shift.

This model seems to be very similar in form to the one in [8], which incorporates temporal period doubling and spatial pattern formation. In [8], the nonlinear period doubling time map at each point is augmented by a linear spatial coupling. In our case, the spatial coupling is nonlinear while the time map is linear with the saturation cutoff.

We are now ready to investigate the dynamics of the model system at hand. As an initial condition, the system is provided with an amplitude fluctuation at a noise level $h_0(\vec{r})=\delta(\vec{r}), -10^{-3}\leqslant\delta(\vec{r})\leqslant10^{-3}$. For the size of an excitation, we set $R=4$, $h_0=7$. With this approach, we actually set the size of an excitation from the onset. We solve the model equation on a square plane with periodic boundary conditions. The coupling constant α serves here as a control parameter of the dynamics.

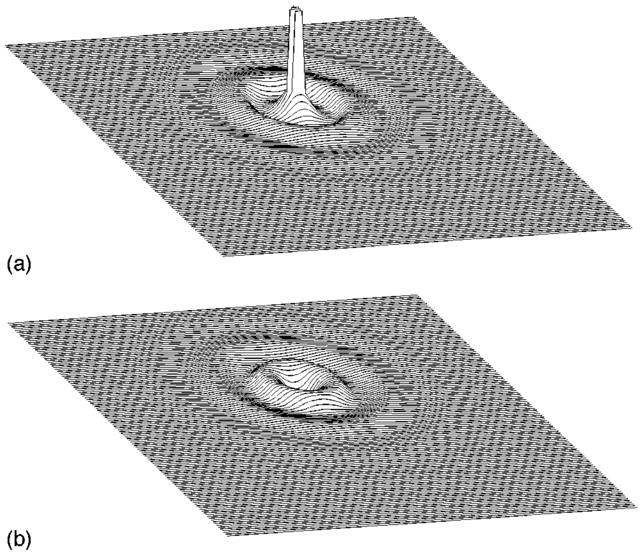

FIG. 2. A three-dimensional perspective of an oscillon of the model system ($\alpha = 13.2$). The oscillon oscillates between (a) the peak and (b) the crater.

We start with an oscillon observed for $\alpha = 13.2$. After the transition period, the dynamics settles down to a steady state oscillating between the two structures as shown in Figs. 2(a), 2(b), where we see a three-dimensional perspective of an isolated excitation. We see clearly that the excitation oscillates with period $\triangle n = 2$ or at frequency $f = 1/2$, on one cycle it forms a peak and on the next a crater. This is the characteristic of an oscillon as reported experimentally in a layer of vibrated sand. The oscillons are found stable in the parameter range of $13.1 \leqslant \alpha \leqslant 14.122$.

Next, we want to investigate the nucleation process of an oscillon, as has been reported experimentally [2]. For convenience we now use a shadow graph to illustrate an oscillon, where a peak of an oscillon is represented by a small circular white spot and a crater by a black spot. The same oscillon of Fig. 2 is shown in Fig. 3(a), where the white spot represents the peak of an oscillon, which oscillates between white and black. Let us now increase the coupling constant above the oscillon's stability boundary to see what would happen to the oscillon. Figure 3(b) shows the scene after 460 time periods after α has been raised to 14.123, where we observe the nucleation process of oscillons. We find that four oscillons have been nucleated around the original oscillon and four additional ones are being nucleated. Specifically, the model shows that oscillons grow by nucleating excitations of opposite phase at their outer edges. The same phenomenology has been reported in experiments [2].

The physical implication of the nucleation is rather significant. Most of all, it implies that a global pattern characteristic to the system is constructed not by an instability of a linear mode, but rather by filling the space with the characteristic localized excitations. It then helps one to understand the occurrence of a variety of patterns including global stripes, oscillon lattice, oscillon chains, and oscillon pairs observed in experiments in association with the oscillons.

The bifurcation diagram of this model system is presented in Fig. 4. For α increasing from 0 to the critical value of 14.6, the randomly distributed initial fluctuation decays into flat surface and there are no patterns organized. As α exceeds the critical value, a global pattern emerges after a long transition period. Figure 5(a) shows the pattern not yet fully grown during the transition period. After this transition period, the system settles down to the global stripes pattern as shown in Fig. 5(b). We see that individual oscillons in Fig. 5(a) during the transition period are connected together to form the stripes in Fig. 5(b). The black and white stripes oscillate with frequency $f = 1/2$, a white band in one cycle turns into a black one in the next cycle, and so on. This result gives clear evidence that the stripes are constructed by individual oscillons. The stripes have grain boundaries in general as seen in Fig. 5(b). As α increases further, the stripes get more curved and defected, yielding more grain boundaries with disorder.

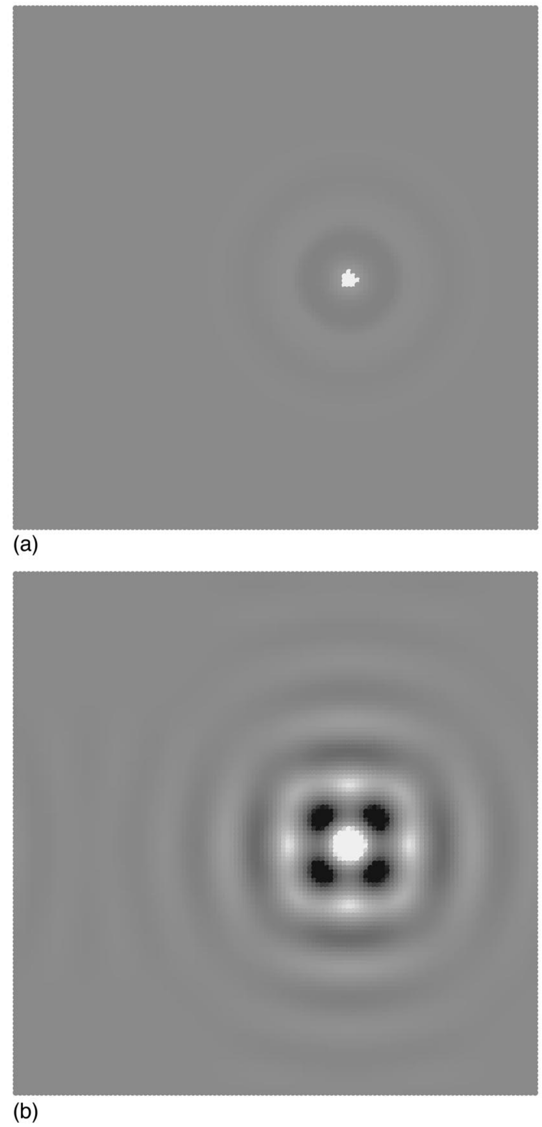

FIG. 3. (a) Shadow graph of Fig. 2(a). (b) The nucleation of the oscillon. Four are nucleated and an additional four are being nucleated.

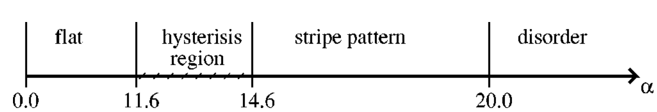

FIG. 4. Bifurcation diagram of the model.

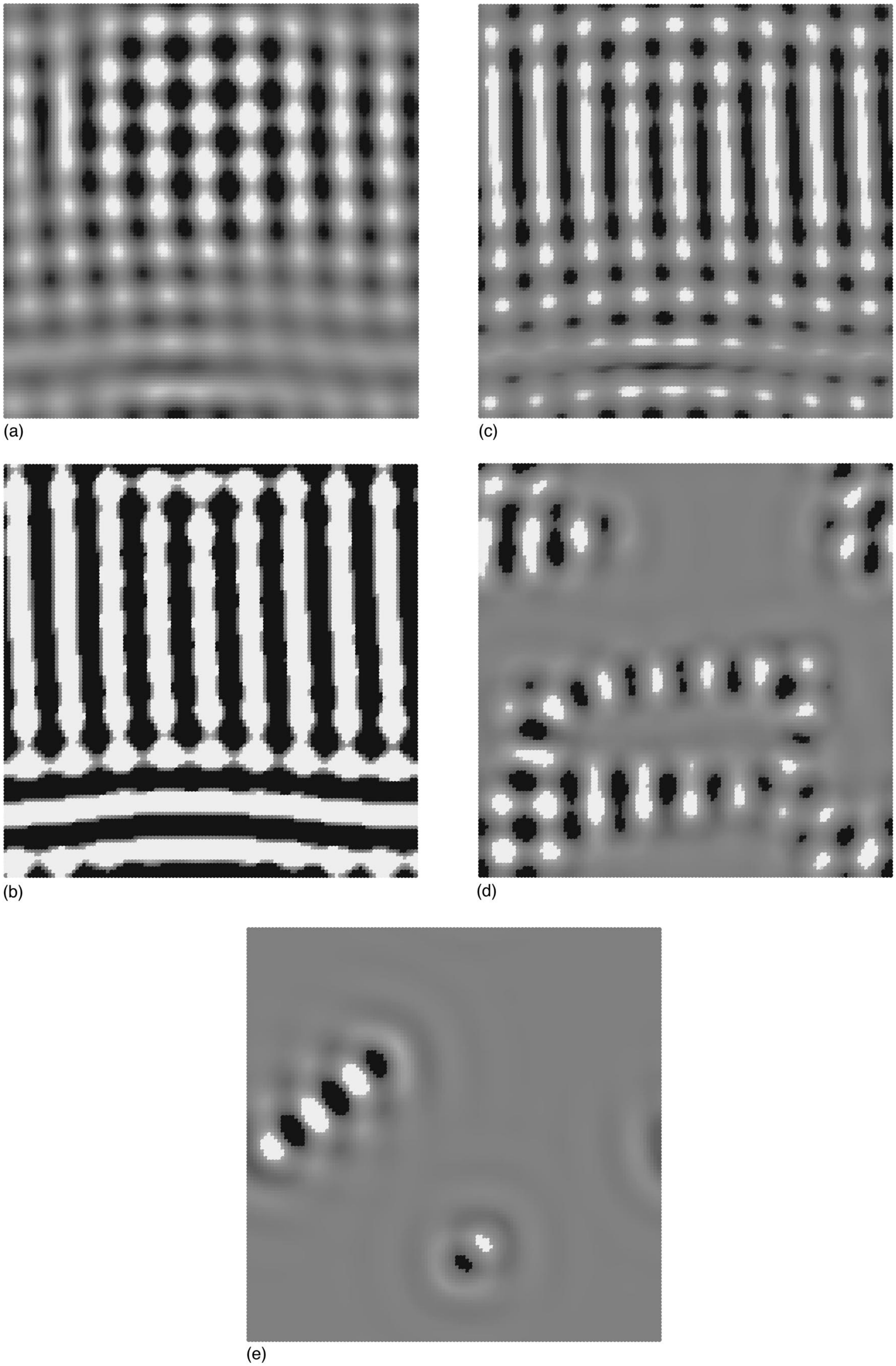

FIG. 5. Dynamics of the model. (a) Oscillons being nucleated during the transition period ($\alpha = 15.0$). (b) Global stripes after the transition period ($\alpha = 15.0$). (c) The global stripes are disintegrated into an oscillon lattice and oscillon chains. (d) Oscillon chains separated from the oscillon lattice. As the flat layer invades, small chains and individual oscillons separate from the lattice. (e) A steady state of an oscillon chain coexisting with a pair of oscillons ($\alpha = 13.3$).

Let us now decrease α below the critical point. We find that the system does not recover the flat space as it was before but instead exhibits hysteresis. We observe the following. First, as the global stripes become unstable, they are gradually disintegrated into localized stripes as well as the oscillon lattices as shown in Fig. 5(c). Second, as time goes on, we see chains of oscillons and individual oscillons separated from the lattice [Fig. 5(d)]. As the flat layer invades, small chains and individual oscillons [cf. Fig. 2] separate from the lattice. Figure 5(e) shows a steady state of a chain of oscillons coexisting with a pair of oscillons at $\alpha=13.3$. They oscillate at frequency $f=1/2$ while the gray background oscillates with $f=1$.

In summary, we have constructed a dynamical model for a qualitative study of the birth of oscillons. In the absence thus far, to our knowledge, of any deterministic equations for the oscillons, our model is discrete in time but continuous in space, and is based on the nearest pattern interaction approximation as defined in the text. The model admits an oscillon solution and features the nucleation process of oscillons. This implies that a global pattern is constructed by oscillons and thus explains why the global stripes are disintegrated featuring oscillon lattices, chains, pairs, and individual oscillons.

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